1) Find the domain of the vector function:

a) $\mathbf{r}(t)=\left\langle \frac{1}{t+1},\frac{t}{2},-3t \right\rangle$

b) $\mathbf{r}(t)=\left\langle \sqrt{4-t^2},t^2,-6t \right\rangle$

c) $\mathbf{r}(t)=\mathbf{F}(t)\times\mathbf{G}(t)$, where $\mathbf{F}(t)=\sin t\ \mathbf{i}+\cos t\ \mathbf{j}$ and $\mathbf{G}(t)=\sin t\ \mathbf{j}+\cos t\ \mathbf{k}$

d) $\mathbf{r}(t)=\mathbf{F}(t)\times\mathbf{G}(t)$, where $\mathbf{F}(t)=t^3\ \mathbf{i}-t\ \mathbf{j}+t\ \mathbf{k}$ and $\mathbf{G}(t)=\sqrt[3]{t}\ \mathbf{i}+\frac{1}{t+1}\ \mathbf{j}+(t+2)\ \mathbf{k}$

a) $\boxed{(-\infty,-1)\cup(-1,\infty)}$

b) $\boxed{[-2,2]}$

c) $\boxed{(-\infty,\infty)}$

d) $\boxed{(-\infty,-1)\cup(-1,\infty)}$

2) Find the limit: (Use L'Hospital's Rule when needed.)

a) $\lim_{t\to 0^+}\langle \cos t,\sin t,t\ln t\rangle$

b) $\lim_{t\to 0}\left\langle \frac{e^t-1}{t},\frac{\sqrt{1+t}-1}{t},\frac{3}{1+t} \right\rangle$

c) $\lim_{t\to\infty}\left(\tan^{-1}t\ \mathbf{i}+e^{-2t}\ \mathbf{j}+\frac{\ln t}{t}\ \mathbf{k} \right)$

d) $\lim_{t\to\infty}\left(e^{-t}\ \mathbf{i}+\frac{1}{t}\ \mathbf{j}+\frac{t}{t^2+1}\ \mathbf{k} \right)$

a) $\boxed{\langle 1,0,0\rangle}$

b) $\boxed{\left\langle 1,\frac{1}{2},3 \right\rangle}$

c) $\boxed{\left\langle \frac{\pi}{2},0,0 \right\rangle}$

d) $\boxed{\langle 0,0,0\rangle}$

3) Evaluate (if possible) the vector function $\mathbf{r}(t)=\left\langle \ln t, \frac{1}{t}, 3t \right\rangle$ at each given value of t.

a) $\mathbf{r}(2)$
b) $\mathbf{r}(-3)$
c) $\mathbf{r}(t-4)$
d) $\mathbf{r}(1+\Delta t)-\mathbf{r}(1)$

a) $\boxed{\left\langle \ln 2, \frac{1}{2}, 6 \right\rangle}$
b) $\boxed{\text{Not Defined}}$
c) $\boxed{\left\langle \ln(t-4), \frac{1}{t-4}, 3(t-4) \right\rangle}$
d) $\boxed{\left\langle \ln(1+\Delta t), \frac{-\Delta t}{1+\Delta t}, 3\Delta t \right\rangle}$

4) Find $\|\mathbf{r}(t)\|$ if $\mathbf{r}(t)=\left\langle \sqrt{t}, 3t, -4t \right\rangle$.

$$\boxed{\sqrt{t(1+25t)}}$$

5) Represent the line segment from $P(0,2,-1)$ to $Q(4,7,2)$ by a vector function and by a set of parametric equations.

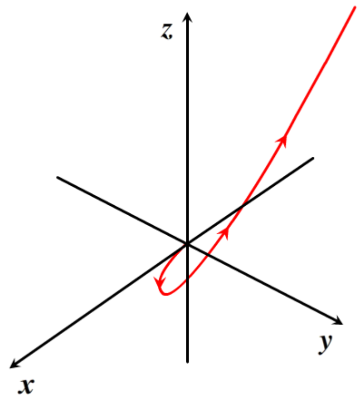
(Answers may vary)

$$\mathbf{r}(t)=\left\langle 4t, 2+5t, -1+3t \right\rangle,\ 0\le t\le 1$$

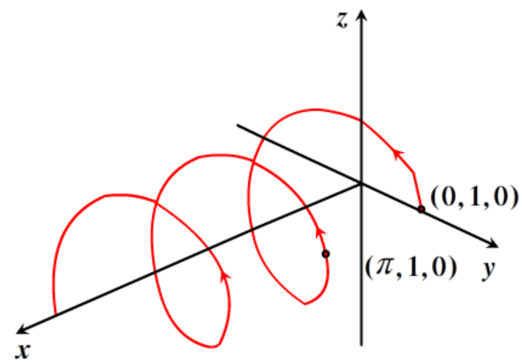
$$x=4t,\ \ y=2+5t,\ \ z=-1+3t\ \ 0\le t\le 1$$

6) Sketch the curve with the given vector function. Indicate with an arrow the direction in which t increases.

$\mathbf{r}(t) = \langle t^2, t^4, t^6 \rangle$



$\mathbf{r}(t) = \langle t, \cos 2t, \sin 2t \rangle$



7) Show that the curve with parametric equation $x = t\cos t$, $y = t\sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$.

$$\boxed{x^2 + y^2 = t^2\cos^2 t + t^2\sin^2 t = t^2 = z^2}$$

8) Find a vector function that represents the curve of intersection of the two surfaces: the cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

$$\boxed{\langle 2\cos t, 2\sin t, 4\cos t\sin t \rangle, \quad 0 \le t \le 2\pi}$$

9) Find a vector function that represents the curve of intersection of the two surfaces: the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

$$\boxed{\left\langle t, \frac{1}{2}(t^2 - 1), \frac{1}{2}(t^2 + 1) \right\rangle}$$

10) Is the vector function $\mathbf{r}(t) = \begin{cases} \mathbf{i} + \mathbf{j} & t \geq 2 \\ -\mathbf{i} + \mathbf{j} & t < 2 \end{cases}$ continuous at $t = 2$?

$$\boxed{\text{No, limit as } t \to 2 \text{ does not exist.}}$$

11) Two particles travel along the space curves $\mathbf{r}(t) = \left\langle t^2, 7t - 12, t^2 \right\rangle$ and $\mathbf{u}(t) = \left\langle 4t - 3, t^2, 5t - 6 \right\rangle$. A collision will occur at the point of intersection if both particles are at the point of intersection at the same time.

a) At what times do the particles paths intersect?
b) At what time and point do the particles collide?

a) $\boxed{t = 1,\ t = 3}$
b) $\boxed{t = 3,\ (9, 9, 9)}$